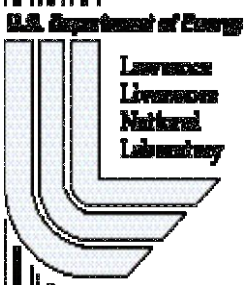
Report: LLNL-TR-2026194

How Accurate are Multi-Group Neutron Transport Codes?

by
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November 22, 2026

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Overview

Today our particle transport codes are extremely well written and very efficient for use on today's computers. The one fault that I see and will address in this paper: **The results from these multi-group codes can be no more accurate than the nuclear and atomic data that they use. Without good data they are in a GARBAGE IN = GARBAGE OUT situation.** In this paper I assume that after almost 60 years of use everyone reading this paper is using nuclear and atomic data in the ENDF format, as uniquely defined in what I refer to as the ENDF bible, ENDF-102 [1]. In this paper I will only present an overview of several effects that effect the accuracy of the data; each of these effects I have already described in detail in other papers. Doppler broadening [2], Self-Shielding, and multi-group cross section models [3] These papers [2, 3] were directed at nuclear data evaluators and producers. Here in this paper my emphasis is to help multi-group code users be sure they use the BEST nuclear data for their applications, from the variety of options that they may to presented to users by nuclear data processors.

For the last almost 60 years I have worked on ENDF/B data since my first post after graduate school in 1967. By now the current ENDF/B-VIII.1 [4] data more than meets our thermal reactor needs. However, the ENDF/B data released to the public for use is far from the data needed and used by multi-group codes. Here I will attempt to address these differences and illustrate the enormous differences between ENDF/B as distributed by nuclear data centers around the World which is all "cold" 0 Kelvin in the center-of-mass frame of reference and the temperature dependent data in the Laboratory frame of reference, which is used by our codes, i.e., the nuclear data must be **Doppler broadened**. This temperature dependence must be addressed for all nuclear transport codes, continuous energy or multi-group. For use with multi-group codes there is an additional layer of approximations in going from continuous energy data to multi-group data. This is where I see a potential weakness: in order to define multi-group constants, they must be weighted by the particle flux. But this sounds like an impossible Catch-22 situation, because it is the particle flux we want to calculate, i.e., we must deal with Resonance **self-shielding**.

Compared to the cold (0 Kelvin), unshielded data the effect of the combination of Doppler broadening and self-shielding the multi-group data we need for use in multi-group codes can result in differences in the multi-group nuclear data of a factor of over 100 – not 100% - a factor of over 100. This paper is designed for **Caveat Emptor = let the user beware**. As multi-group code users make sure you are using the most appropriate nuclear multi-group data for use in your applications: I will keep repeating: Even with a perfect nuclear transport code **failure to use the best nuclear data you can find yourself in a GARBAGE IN = GARBAGE OUT situation.**

Prologue

I am now approaching 87 years of age, and I am trying to document and pass on the following generations what I have learned. This paper is one of a series that I am writing to document my almost 60 years of experience with Neutron Transport calculations and ENDF nuclear and atomic data. My doctorate thesis was on neutron integral transport theory [5, 6]. By the time I was close to finishing my thesis (mid-1960) I realized that the particle transport codes of the day were much more accurate than the then available nuclear and atomic data. It is so important to realize that:

regardless of how accurate or even perfect our application codes may be to transport particles, without accurate nuclear data we are in a "Garbage In = Garbage Out" situation.

At the time (1967) I assumed I could be of the most help working to improve the nuclear and atomic data used by our particle transport codes. This led me to accept as my first post out of graduate school, which was a position at the newly created National Nuclear Data Center (NNDC), Brookhaven National Laboratory, where I worked on ENDF data from 1967 to 1972. During those years I was the head of the computer unit of NNDC, assigned to develop computer codes to pre-process, view and test ENDF/B data. By 1972 we were prepared to release ENDF/B-III, which was the first public release of ENDF/B: I published it in graphic form [7] which was the first time we actually saw all of the ENDF data; up until that time ENDF/B was restricted by the AEC to only domestic use. By 1972 I felt that I had served my time and wanted to return to my first love, nuclear and atomic transport codes, and I moved to Lawrence Livermore National Laboratory (LLNL), to work on the TART [8] Monte Carlo particle transport code.

Since then, I have continued to support the ENDF effort without any official position or monetary compensation, because I realized how important accurate nuclear data is for use in use in our Engineering applications. It is so important I will repeat **regardless of how accurate or even perfect our application codes may be to transport particles, without accurate nuclear data we are in a "Garbage In = Garbage Out" situation.**

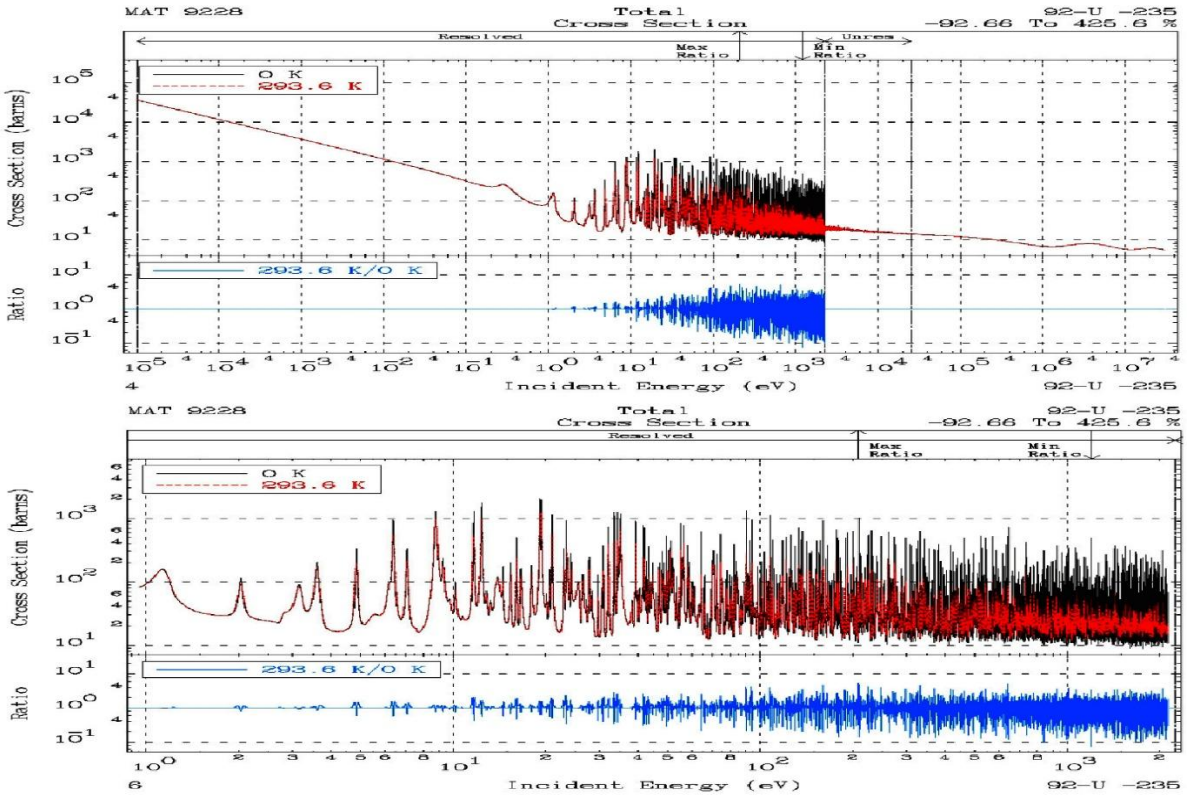
During these past 60 years we have seen a tremendous improvement in the nuclear data available for our use in our **Engineering Applications**. This was to a large degree because of the ENDF format and conventions uniquely defined in what I call the **ENDF Bible, ENDF-102** [1]; it has served as a bridge between evaluators, who produced the data, and engineers who use the data, and most importantly is that users test the data, and supply feedback to the evaluators to further improve the next version of ENDF/B. The close coupling has always been imperative through each version of ENDF/B; today the most recent release is ENDF/B-VIII.1.

How Accurate are our Multi-Group Transport Codes?

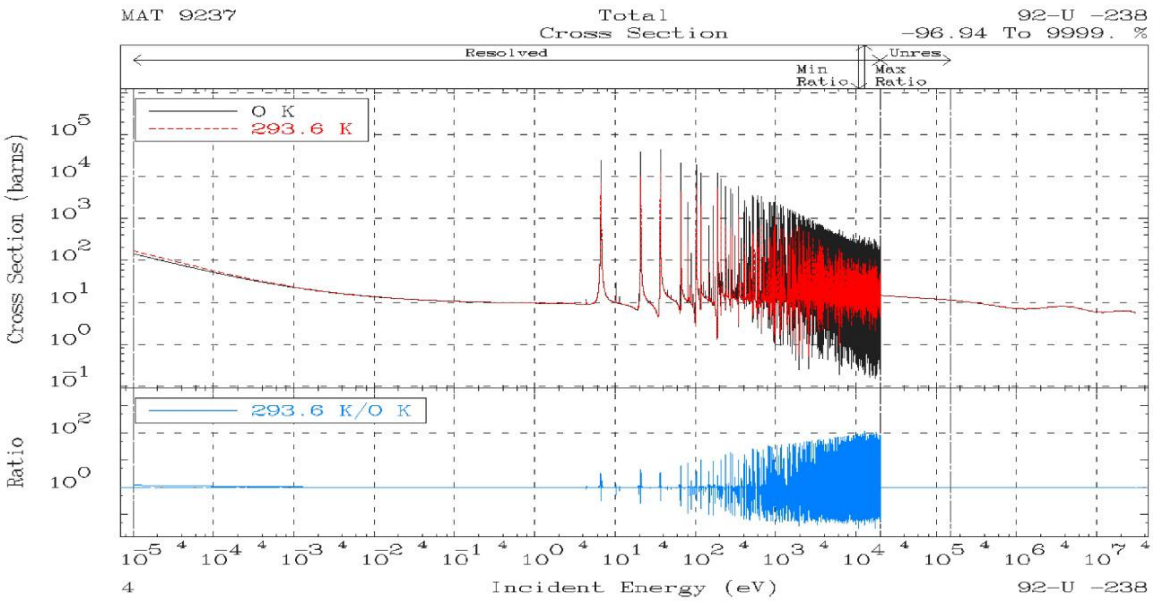
This report is aimed at helping data users of nuclear and atomic data in their application codes. Particularly I ask the question: How accurate are our multi-group codes? In an attempt to answer this question, I will use the TART [8] and MCNP [9] Monte Carlo transport codes to illustrate how a variety of models for the Uranium (U235 and U238) cross sections effect one theoretical very simple system in detail and also present an overview of K-eff for a variety of real slow fission systems.

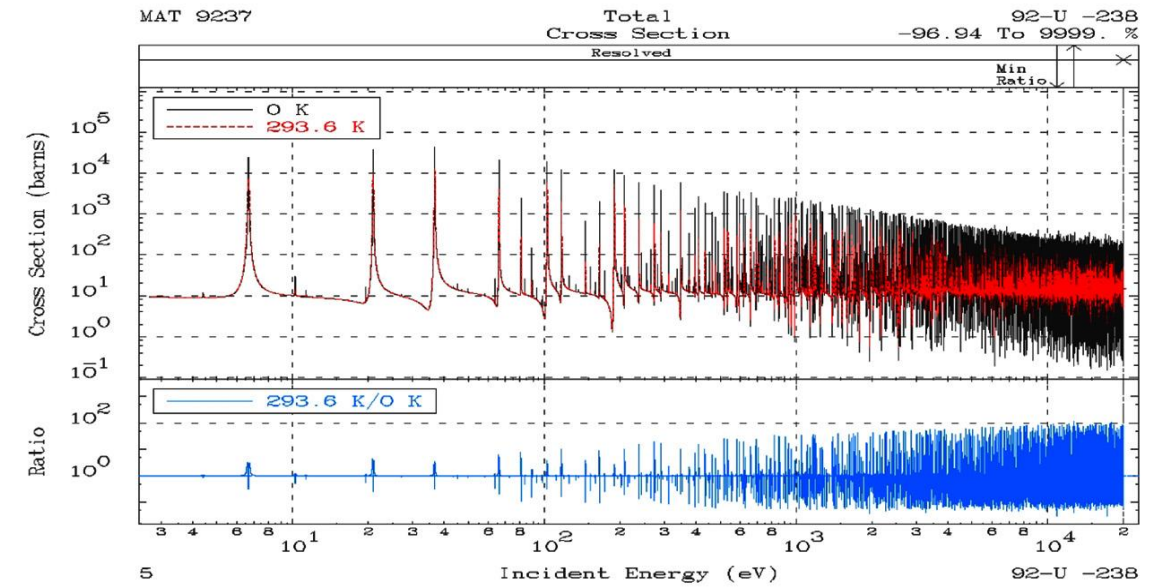
Doppler Broadening

The first thing for users to understand is that all of the ENDF/B data supplied by evaluators is in the form on resonance parameters and tabulated data at 0 Kelvin in the center-of-mass system [1, 4]. Before you can use it in applications it must first be converted to temperature dependent data in the Laboratory frame of reference. While working at NNDC (1967-72) I created my SIGMA1 method of Doppler broadening [10, 11], to broaden tabulated linearly interpolable cross sections, as part of my ENDF Pre-Processing codes, PREPRO [12], and I freely gave these codes to anyone who was interested. To illustrate the magnitude of the changes that Doppler broadening can have on nuclear data, the below figure compares, U235 Total, at 0 and 293.6 K (20 C). Even over this relatively small temperature range (0 to 293.6 Kelvin) the U235 total: peaks can reduce to only 8% (-92% decrease) and minima between resonances are increased by up to over a factor of 4 (425 %). **Doppler broadening obviously plays an enormous role in defining the cross sections you need for applications.**



U235 is the most important isotope to make thermal fission reactors critical, but in today's reactors it is only about 3% to 5% of the Uranium; most of the Uranium is U238. So below I show similar results for U238; here I again compare 0 Kelvin and 293.6 Kelvin (20 Celsius), U238 total. For U238 Doppler broadening plays an even larger effect than in U235. Compared to U235, the capture resonances in U238 are narrower and widely seperated (isolated). Here the resonance peaks are decreased to less than 4 % (over 96% decrease) and minima between resonances are increased by over a factor of 20 (FACTOR of 20; not 20%). Need I repeat: **Doppler broadening obviously plays an enormous role in defining the cross sections you need for applications.**



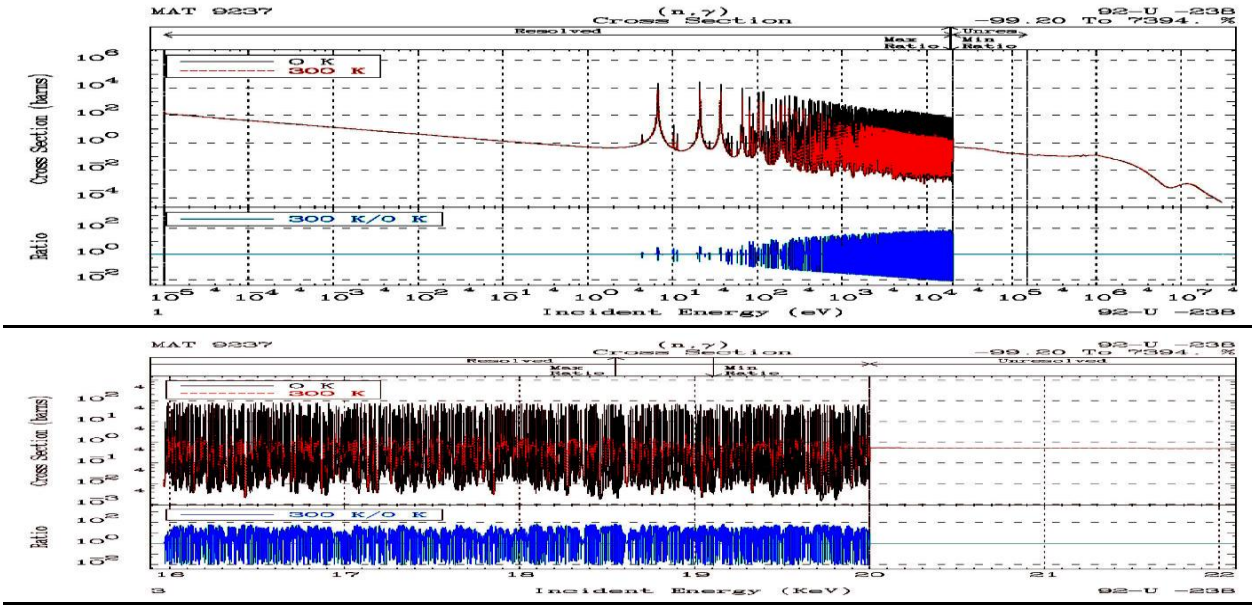


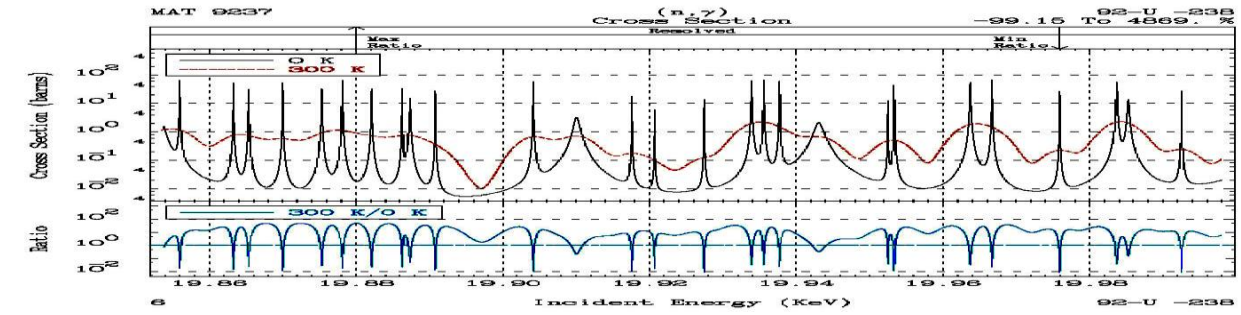
U-238 Capture: at 0 and 300 Kelvin

The below figures illustrate the enormous effect that Doppler Broadening has on the U238 Capture Cross Section at two temperatures: 0 and 300 Kelvin (room temperature). The below three figures show the U238 capture starting from an overview and zooming in to show details of the very narrow capture resonances near the upper limit of the RESOLVED resonance region at 20 keV.

First note the UNRESOLVED resonance region. This is what we expect from ENDF data. By definition [1] we do not uniquely know the unresolved energy dependent cross sections, and in plots of the data dating back to my publication of ENDF/B-III [7] we traditionally show the UNSHIELDED average cross section. In a properly defined ENDF evaluation **the UNRESOLVED REGION is smooth and INDEPENDENT of temperature.** [2]. Note, the expected discontinuity between the resolved **energy dependent** and unresolved **averages**. Let me repeat: the ENDF defined cross section is energy dependent in resolved and unshielded average in the unresolved.

Next, **note the ENORMOUS effect that Doppler broadening has on these very narrow U238 capture resonances**. The peak at resonances and valley between resonances change by almost **a factor of 100 – not 100% - a factor of 100!** The below plot shows that virtually all of the narrow resonances shown here up to 20 keV, disappear even at room temperature (300 K)




MAT 9237
(n,γ)
Cross Section
92—U —238
Resolved
—99.15 To 4889. %
Max Ratio
Min Ratio
0 K
300 K
300 K/0 K
Cross Section (barns)
Ratio
19.86
19.88
19.90
19.92
19.94
19.96
19.98
Incident Energy (KeV)
92—U —238

Multi-Group Equations and Self-Shielding

One obvious problem with the Multi-Group method is that we have to "guess" the answer in order to define the multi-group constants, and our "guess", even when using many groups (in my TART Monte Caro code [8] I use 616 groups), can still have a major impact on the accuracy of our answers.

Let's look at the source of the problem. The following applies to any geometry, but for simplicity, I will illustrate results starting from the time independent, linear transport equation, in planar geometry with continuous energy, and I will then explicitly describe the six models available for testing in TART [8] for the **continuous**, **multi-group** and **multi-band** cross sections,

$\mu\ \partial\Phi(E,z,\mu)/\partial z + \Sigma t(E,z)*\Phi(E,z,\mu) = R(E,z,\mu)$; here $R(E,z,\mu)$ is the slowing down and sources.

The multi-group equations are obtained by integrating over adjacent energy ranges, and usually also integration over spatial regions, and direction, to define multi-group constants,

$\mu\ \partial < \Phi(E,z,\mu)dE > /\partial z + <\Sigma t(E,z)*\Phi(E,z,\mu)dE> = <R(E,z,\mu)dE>$, or, equivalently,

$\mu\ \partial\Phi g(z,\mu)/\partial z + \Sigma tg(z)*\Phi g(z,\mu) = Rg(z,\mu)$

Where the group averaged total cross section is defined as **the group averaged ratio of reactions to flux,**

$$\Sigma tg(z) = \frac{<\Sigma t(E,z)*\Phi(E,z,\mu)dE)>}{<\Phi(E,z,\mu)dE>} = \frac{\text{Reactions}}{\text{Flux}}$$

Our problem is that in order to define our multi-group cross sections, $\Sigma tg(z)$, we must "guess" at the **energy dependence** of the flux, $\Phi(E,z,\mu)$, which is the answer we are trying to find; sounds like an impossible "Catch-22" situation. Less obvious we are integrating not only over energy, but usually **also over space and direction**. Generally, the energy dependent flux will be approximated by the product of two terms: **an energy dependent spectrum**, e.g., Maxwellian at low energy, 1/E in the slowing down range, and a fission and even possibly fusion source at higher energies, and **a cross section dependent,** self-shielding factor. For the slowing down spectrum here I assume it is "smooth" in energy, which implies the **narrow resonance** (NR) approximation (an additional complication I will only briefly address here in this paper). For a sufficiently large number of groups the assumption of the energy dependent term plays only a minor role, and since it appears in both numerator and denominator **its normalization plays little or no role at all**.

Unfortunately, the same cannot be said about the cross section dependent self-shielding factor. Here the effect is persistent even when using many groups, indeed it plays a role as long as the width of a group is large compared to the width of resonances in the total cross section. We can see this by returning to our original energy dependent equation and assuming,

$\Phi(E,z,\mu) \sim \Phi(E,\mu)*Exp[K*z]$, similar to a Laplace transform or Case's method

$\{\mu K + \Sigma t(E,z)\}\Phi(E,\mu) = R(E,\mu)$

$$\Phi(E,\mu) = \frac{R(E,\mu)}{\{\mu K + \Sigma t(E,z)\}}$$

$R(E,\mu)$ is an energy and direction dependent secondary term

here the denominator is the self-shielding factor.

We can see here that the neutron flux is inversely related to the total cross section, plus a spatially and direction dependent term, μK. The flux (the interpretation we use in our Monte Carlo codes) is the distance travelled by the neutrons normalized per unit time, space, energy and direction; which definition helps us to understand the above inverse relationship to the total cross section, e.g., doubling the total cross section will half the distance travelled by the neutrons, and hence result in half the flux. **This is referred to as "self-shielding" because it is the variation in the total cross section, itself, that causes variation in the flux;** it is this flux that we want to calculate using our multi-group equations, but as we can see here it is also the flux that we must "guess" in order to define our multi-group constants, i.e., Catch-22.

The most commonly used self-shielding models start from our above equation, and assume we have an **infinite, homogeneous** medium, i.e., ignore the μK term,

$$\Phi(E) = \frac{R(E)}{\{\Sigma t(E,z)\}}$$; note, here I have omitted directional dependence of flux, $\Phi(E)$ and $R(E)$

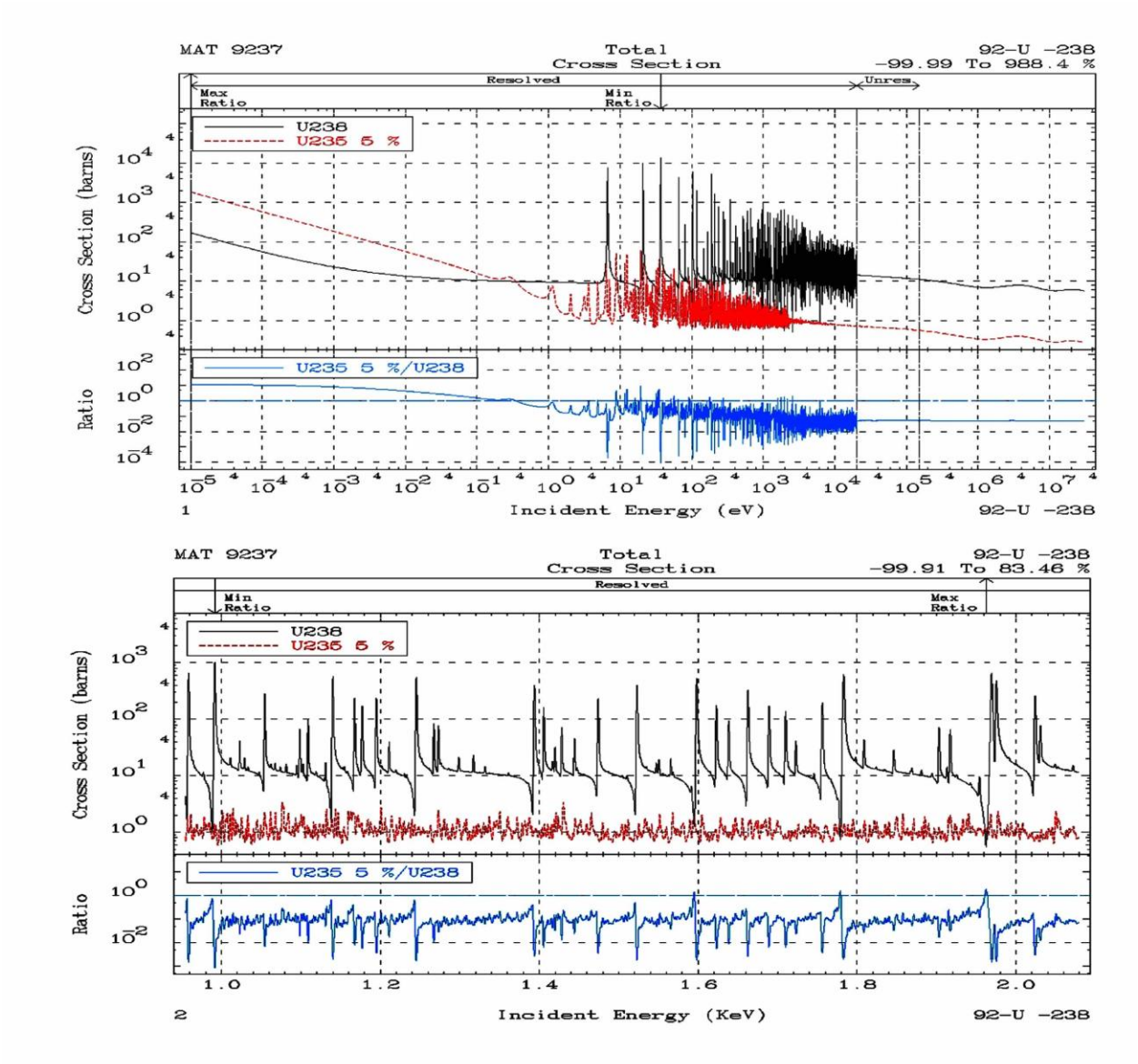
Bonderenko Self-Shielding

An additional complication is that here the total cross section in the denominator is not the total of each evaluation (which is application independent), but rather it is the total of whatever mix of materials we are using in each spatial zone we are averaging over (obviously application dependent). Historically due to limited computer resources we wanted to only define application independent multi-group libraries, so another approximation is introduced; the **Bondarenko**, or partial shielding, where we assume that the total cross section for any mixture can be defined as that of each evaluation, plus a second cross section, Σ_0, which can be any value between zero (Totally Shielded) to infinity (Unshielded or infinitely dilute), cross sections,

$$\Sigma tg(z) = \frac{<\Sigma t(E,z)* R(E)\, dE/\{\Sigma t(E,z) + \Sigma_0\}dE>}{<R(E)\, dE/\{\Sigma t(E,z) + \Sigma_0\}dE>}$$; $\Sigma tg(z)$ constant within spatial zone (z)

The below figure shows Uranium total cross sections; U238 and 5% U235. We can see that one of these cross sections can hardly be approximated as constant relative to the other. The Bonderenko approximation was necessary 50 years ago, but with today's computers this approximation is no

longer needed. Today we can afford to define the actual total for each mixture and use this for self-shielding calculations. Failure to use more modern methods can result in very large differences in multi-group constants, and yet today so many nuclear data processing codes, and application codes still use it. Again: **Caveat Emptor = let the user beware! Are you using this in your codes?**



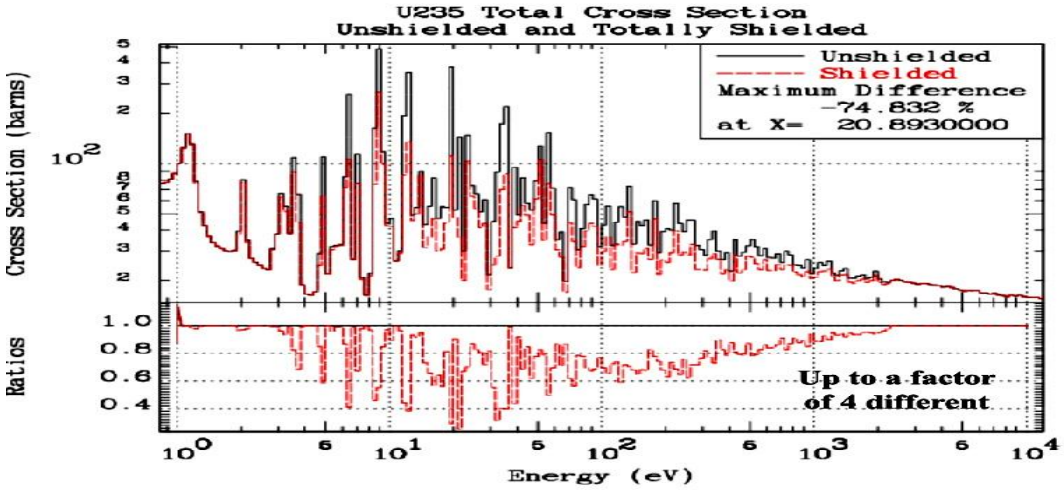
Let's review the explicit approximations used to arrive at this equation,

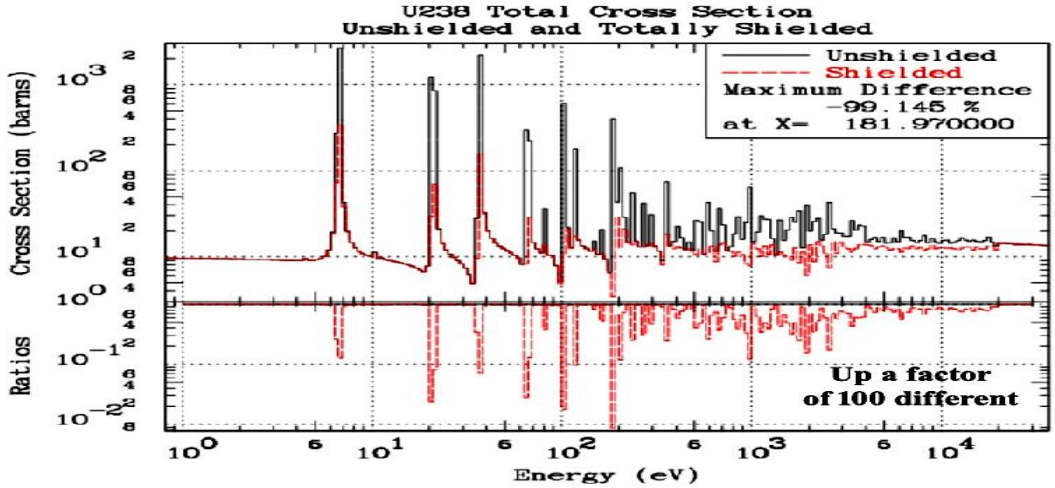
1) Infinite, Homogeneous media; ignore μK.
2) Narrow resonances, which affects the slowing down spectrum, R(E).
3) Resonance structure in each evaluation independent; $\Sigma t(E,z) + \Sigma_0$

In addition, there are less obvious implied approximations.

1) Multi-group constants are constant throughout each spatial region
2) Multi-group constants are independent of direction.

The below plots illustrate the U235 and U238 Total Cross Section using the TART 616 group structure (50 groups per energy decade from 10^{-5} eV to 20 MeV), comparing Unshielded and Totally Shielded cross sections. These plots illustrate that even with 616 groups, the U235 Shielded Total can be up to ~ 75% less than the Unshielded (i.e.,1/4-th of it), and for U238 it can be over 99% less (100 times less). **So the differences, even with 616 groups, due to self-shielding can be ENORMOUS; increasing to 2,020 groups does not improve things by much; e.g., e**ven with 2,020 groups the groups are still wide compared to narrow U235 and U238 resonances.





One very important point to note: if a region of space contains only one mixture of materials, dividing this region into smaller and smaller volumes has no effect on these self-shielding models, so regardless of how fine we zone our geometry, this will not change the definition of the multi-group constants and results shown here.

Cross Section versus Distance to Collision

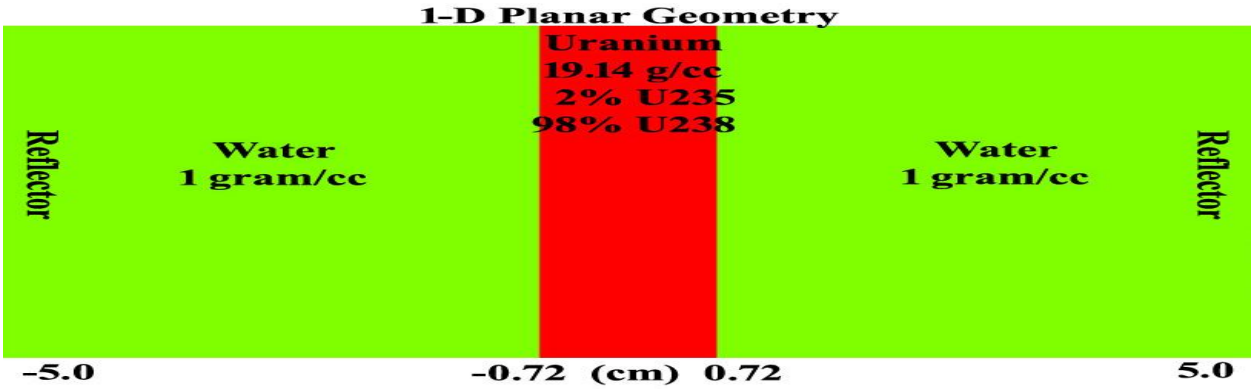
Note that above I only mention calculating the cross section, but in our Monte Carlo codes we sample the distance to collision, not the total cross section,

Distance = - log(random)/ $\Sigma t(E,z)$

Here we are trying to estimate the flux = the distance travelled by the neutrons. This also suggests that when we estimate the flux used as weighting in our multi-group equations, it should also be the distance to collision, or the inverse of the total cross sections, as shown in the above equations.

Simple 1-D Planar Geometry Example

To illustrate results here I calculated a very geometrically simple, theoretical critical system of a 1-D, planar, infinitely repeating array of Uranium layers in Water. I start with this extremely simple system to make it as easy as possible for anyone to replicate these results using your own neutron transport codes, particularly deterministic (multi-group) codes. Here there is no question as to what the "exact K-eff" is; there is no measured value. This is strictly theoretical and all results are based on the same ENDF/B data 293.6 K data, so I assume the BEST K-eff is based on using continuous energy cross sections (almost exactly 1.0) and any deviation from this value is based strictly on the multi-group self-shielding models used here.



I used continuous energy cross sections as well as the TART 616 groups (50 per energy decade from 10^{-5} eV to 20 MeV) to represent the multi-group and multi-band cross sections (2 bands per group). The only difference in the transport calculations was the representation of the neutron cross sections as either continuous or multi-group (histograms), e.g., in all six cases I used the same continuous energy neutrons, geometry and reaction kinematics to describe all secondary neutrons; only the multi-group (histogram), cross sections differed. Today we seem to have FAITH (faith = belief, without proof) that we can calculate K-eff to an accuracy of roughly 0.1% (3 digits),

whereas the below results illustrate that if we do not account for self-shielding the difference between the results using continuous energy cross sections and 616 groups is over 0.025, 2.5%, or over 25 TIMES what we consider acceptable. Let me repeat**: the difference is not 25%; it is 25 TIMES the 0.001 in K_{eff} 0.1%, that we consider acceptable. This difference is solely because of our "guess" for the flux that I used to define the multi-group constants, i.e., solely due to the self-shielding model.**

Note the running times: These are typical, showing that today using continuous energy cross sections is no longer prohibitively expensive, which is why **this is "The Standard BEST Option" default that I use for all of my TART production work**; here I use multi-group and multi-band results solely in the hope that the results can be useful by others/you, the reader, in their multi-group calculations; particularly deterministic calculations**.**

Planar U/Water Criticality Results

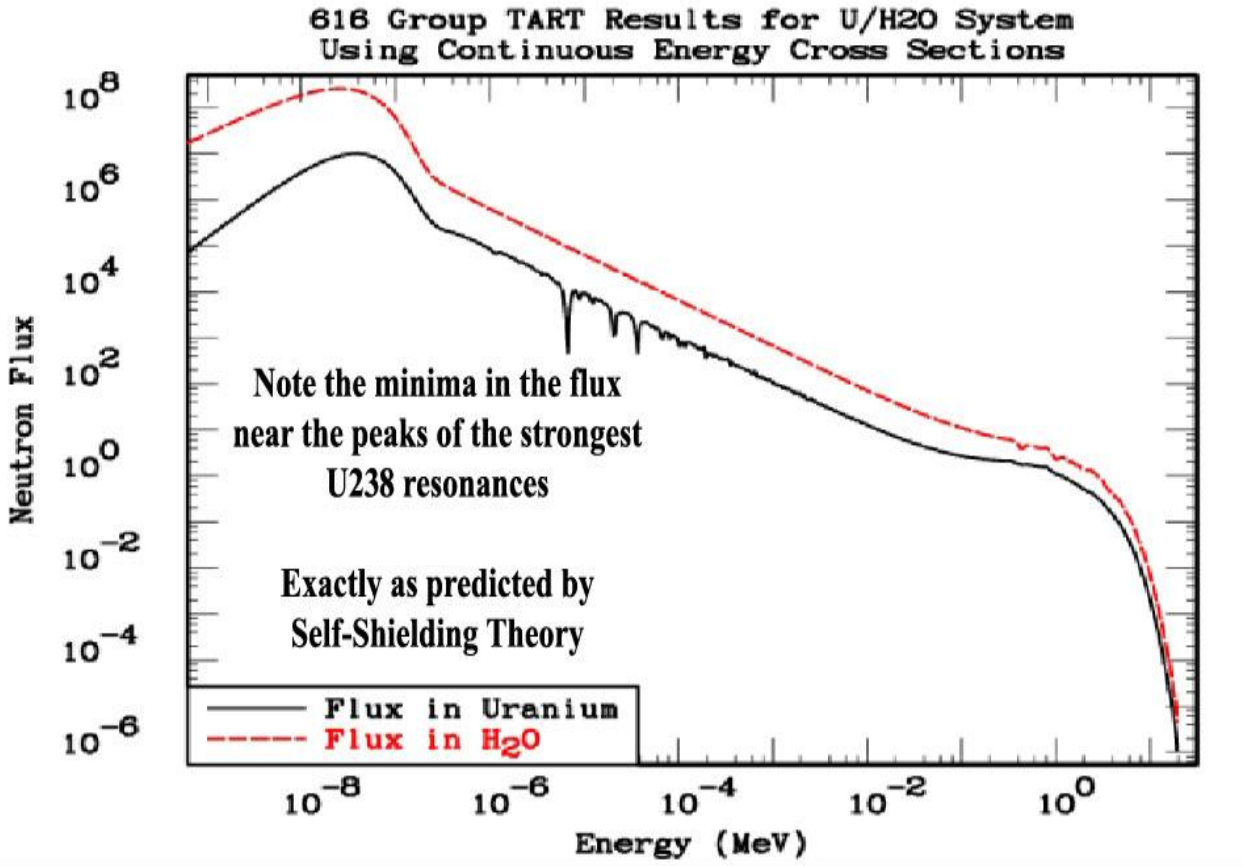
Cross Section Representation	Expected K-eff	Difference in K-eff	Removal Lifetime (Microsec.)	Median Energy (eV)	Seconds
Continuous	0.999924	--------	7.89162D+01	5.00948D-02	5066.660
Multi-Band	1.000980	0.001056	7.90684D+01	5.01097D-02	4404.120
Multi-Group	0.974683	-0.025241	7.70376D+01	5.01225D-02	3837.700
Unshielded	0.974645	-0.025279	7.70372D+01	5.01247D-02	4442.640
Totally Shielded	1.001750	0.001826	7.97356D+01	4.95769D-02	4581.800
Partial Shielded	0.991570	-0.008354	7.86170D+01	4.98773D-02	4512.280

Today we have FAITH (again, belief, without proof) that we can calculate critical systems within roughly 0.1%, i.e., 3-digits of accuracy. But the above results using 616 groups, demonstrates that,

1) Not self-shielding at all leads to differences of ~ 2.5%; **25 TIMES what we assume**.
2) Totally or Partially shielding improves results, but not to ~ 0.1%
3) Only the Multi-Band (in this case only 2 Bands) produces agreement ~ 0.1%.

I should stress that this is for a very simple, 1-D planar, system; far from what we expect to find in the real World, but at least to me this is as simple as I can model a critical system that readers of this report can easily try in their neutron transport criticality computer codes.

Below is a figure showing the neutron flux in the Uranium and Water, separately. The spectrum is more or less what we expect: a fission spectrum at high energy, a 1/E slowing down spectrum at intermediate energies, and a thermal Maxwellian. As it relates to this paper, note the minima in the flux in the Uranium near the peaks of the strongest ^{238}U resonances; exactly as predicted by Self-Shielding theory; it's real. Also, as we expect, there are no such minima in the water, showing the strong spatial dependence of the self-shielding.



The Unresolved Resonance Region

Even codes that use continuous energy cross sections, must account for self-shielding in the ENDF-102 defined unresolved resonance region (URR) [1]. In earlier papers [2, 3] I address this point, using NJOY [19] and MCNP [4], with and without self-shielding in the URR. The following test were performed.

First, we demonstrated the importance of self-shielding in the URR by performing MCNP [9] calculations of 32 critical assemblies with and without URR self-shielding; all these results are based on running an unmodified version of MCNP. The following results are for,

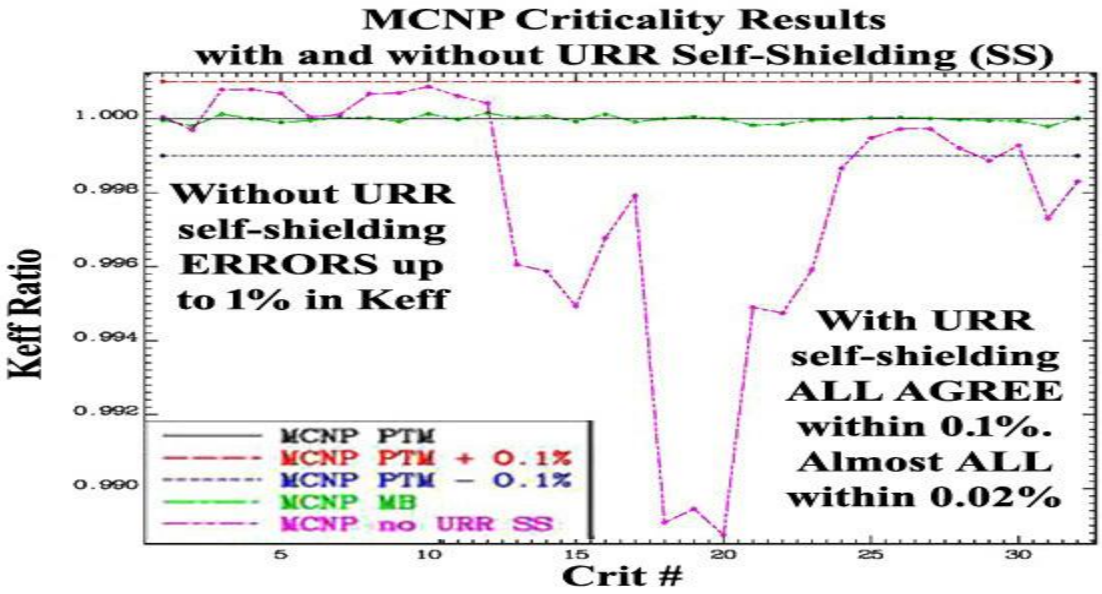
1) Not including URR self-shielding (SS), i.e., using unshielded, or infinitely dilute.
2) Using the Probability Table Method (PTM), with 20 cross section bands [13, 14].
3) Using the Multi-Band Method, using only 2 cross section bands [15, 16, 17, 18].

On the below figure the MCNP PTM +/- 0.1% lines are included merely as eye guides.

In the below figure the crit # corresponds to the numbers in the critical assembly identified in my paper addressing self-shielding in detail [3]. Even without identifying the individual assemblies the below figure clearly shows that,

1) In these cases, unresolved region self-shielding (URR SS) had an important macroscopic effect on the results. **Here we see that if we do not account for URR Self-Shielding the answers can be up to 1% different** from the standard MCNP results without URR Self-Shielding, well outside the accuracy we are attempting to achieve.

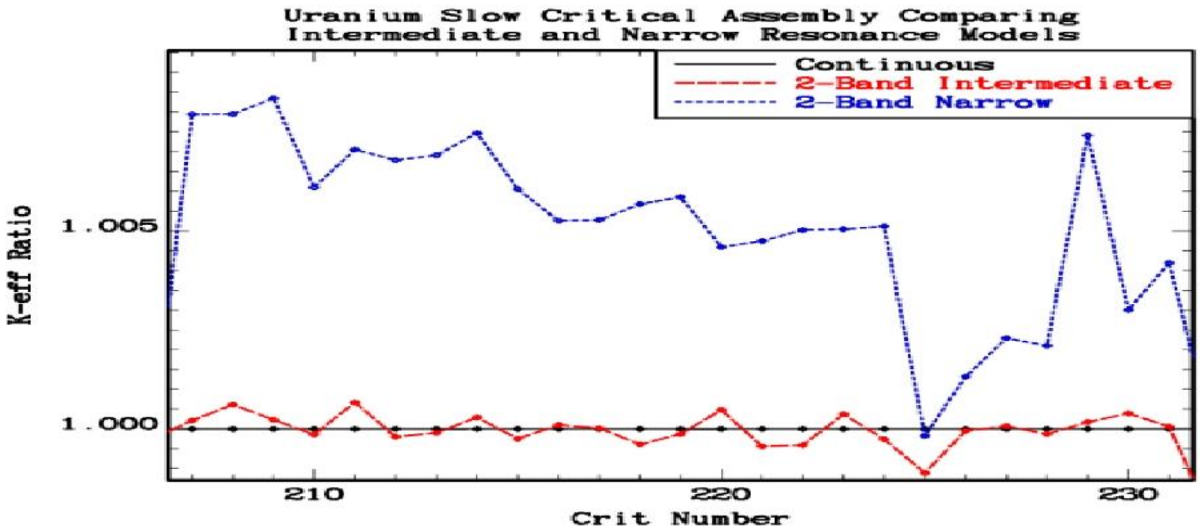
2) In ALL these cases **the results using 20 PTM bands or 2 MB bands are indistinguishable**, well within 0.1% of one another; indeed, almost ALL are within 0.02% of one another, far below the level of agreement we expect/require in our applications.



Narrow, Wide and Intermediate Resonance Approximations

All of the above results are based on the Narrow Resonance (NR) approximation, which assumes after a collision each neutron can "see" the entire range of cross section due to resonances. In the opposite extreme is the Wide Resonance (WR) approximation, assumes after a collision neutrons "see" the same cross section as before the collision. A third possibility is the Intermediate (IR) approximation, which assumes after a collision there is a probability that the neutrons will see both Narrow and Wise resonances each with some probability, the 2 summing to unity. The effect that these three resonance width models are described I detail in [3, 16, 17]. Here I will only include a figure showing that in the few cases where my normal 2-band calculations slightly exceed the 0.001% difference from using continuous energy cross sections, using the Intermediate Resonance (IR) model can eliminate this slight difference. Below is a copy of a small portion of the discussion of Intermediate Resonance (IR) model in [3]. In this paper introducing data users to self-shielding, I judge that it is sufficient to refer readers to [3, 20] if you are interested in Intermediate Resonance (IR) approximation.

Uranium Slow Critical Assemblies: Below is a comparison for the Uranium Slow Critical Assemblies 207 through 231. These are the assemblies where we found (in ref. 3) significant differences using the Narrow Resonance (NR) Model. Below the Narrow Resonance (NR) results are compared to Intermediate Resonance (IR) Model results. The results show that for these assemblies using the Intermediate Resonance (IR) reduces the difference to within 0.1%, except for one case (225 cases) where it is 0.11% (which I judge to be statistically the same as 0.1%).



Multi-Band Method

As we add more and more groups to reduce the effect of energy on results, with fissionable materials we can quickly arrive at the point where it in the variation in cross section that becomes more important than that in the energy. This suggests that further refinement should be based on sub-division based on **Lebesque** integration over ranges in cross section, as opposed to the sub-division on my **Riemann** integration over ranges in energy that we are used to. That is all I do in my multi-band method. I extend the probability table method, that I learned from Leo Levitt [13, 14], in the unresolved energy ranges, to the entire energy range [15, 16, 17, 18]. Leo used 19 cross section ranges, whereas today I use only 2 cross section bands. This simple extension replaces multi-group constants that are constant throughout each spatial zone, independent of space and direction, by the 2 band data that allows for the equivalent multi-group cross section (the sum of 2 band cross sections results) that vary in space and direction within each spatial zone and reproduce the expected normal multi-group results in the unshielded, totally shielded and partially shielded limits.

Conclusion

The title of this paper asks: how accurate are multi-group neutron transport codes? Based on my almost 60 years of experience the answer is: **these codes can be very accurate** as long as they use accurate nuclear data and models to correctly account for temperature (Doppler broadening) and self-shielding.

In the other extreme some currently used models to define multi-group data can result in rubbish results. Even the simplest 1-D theoretical problem I have shown here result in differences in K-eff of up to 2.5%, i.e., 25 time more than the 0.001 accuracy we think we can calculate results. In a self-serving manner let me end by pointing out that of the six multi-group averaging methods shown here only my multi-band (today I always use only 2 cross section bands) produces the single result, K-eff, to within the 0.1% accuracy we assume we can produce, agreeing with the results using the continuous energy data.

References

[1]"**ENDF-6 Formats Manual** Data Formats and Procedures for the Evaluated Nuclear Data File ENDF/B-VI and ENDF/B-VII", Written by the Members of the Cross Sections Evaluation Working Group; Last Revision, Edited by M. Herman and A. Trkov, February 2018.

[2] "**Doppler Broadening** and other Temperature Effects", Lawrence Livermore National Laboratory, LLNL-TR-2005605, April 2025.

[3] **Resonance Self-Shielding: Why it is so Important**", Lawrence Livermore National Laboratory, LLNL-TR-2011238, July 2025.

[4] "**ENDF/B-VIII.0**: The 8th Major Release of the Nuclear Reaction Data Library with CIELO-project Cross Sections, New Standards and Thermal Scattering Data", with D.A.Brown and many others, Nucl. Data Sheets **148** (2018).

[5] "Numerical Solution of the Linear Integral Boltzmann," Dermott E. Cullen, doctoral thesis, Columbia University (1970)

[6] "Numerical Solution of the Linear Integral Boltzmann," Nuc. Sci. and Eng. 53, p. 93-106 (1974).

[7] "ENDF/B Cross Sections," BNL-17100 (ENDF-200) (1972).

[8] **TART 2022**: An Overview of A Coupled Neutron-Photon 3-D, Combinatorial Geometry Time Dependent Monte Carlo Transport Code", Report: LLNL-SM-835527, Code Release: LLNL-CODE-836321, May 2022.

[9] **MCNP** - A General Monte Carlo N-Particle Transport Code, Version 5, Vol. I: Overview and Theory, X-5 Monte Carlo Team, Los Alamos National Laboratory report LA-UR-03-1987 (April 24, 2003).

[10]. Program **SIGMA1** (Version 74-1), Lawrence Livermore Laboratory Report UCID-16426, January 1974.

[11] **SIGMA1** "Exact Doppler Broadening of Evaluated Neutron Cross Section," Trans. Amer. Nuc. Soc. 16, p. 320 (1973).

[12] **PREPRO 2023:** 2023 ENDF/B Pre-processing Codes (ENDF/B-VIII.0 Improved Precision)", IAEA, NDS-0241, Nuclear Data Section (NDS), IAEA, Vienna, Austria, June 6, 2023

[13] L.B. Levitt: "The probability table method for treating unresolved resonances in Monte Carlo calculations", BNL-50387 (ENDF-187), May 1972.

[14] L.B. Levitt: "The probability table method for treating unresolved resonances in Monte Carlo calculations", Nucl. Sci. Eng. **49**, No 4, 450-457 (1972).

[15] "The Multi-Band Method in Neutron and Photon Transport Problems," A-Division Seminar, Lawrence Livermore Laboratory, Livermore, CA, June 1978.

[16] "Multiband Calculations in Neutron and Photon Transport Problems," B-Division Seminar, Lawrence Livermore Laboratory, Livermore, CA, September 1978.

[17] D.E. Cullen, Yigal Ronen (Ed.): "**Nuclear Cross Section Processing**", Handbook of Nuclear Reactor Calculation, vol. I, CRC Press, Inc., Boca Raton, Florida (1986).

[18] D.E.Cullen, "**Nuclear Data Preparation**", pp. 279-425, Vol. 1, in: "The Handbook of Nuclear Engineering", Springer Publishing, NY, NY (2010).

[19] R.E. MacFarlane, etal,, The **NJOY** Nuclear Data Processing System, Version 2012, LA-UR-12-27079, August 2013.

[20] Goldstein, R., "Temperature-Dependent Intermediate Neutron Resonance Integrals", Nucl. Sci. Eng. **48**, 248-254 (1972).

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